San Diego Math League Euler Olympiad
March 29, 2008

1. Find all ordered pairs of integers (m, n) such that

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{7}.$$

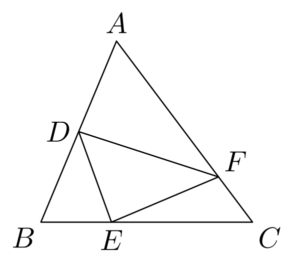
2. Find the sum of the first 55 terms of the sequence

$$\binom{0}{0}, \quad \binom{1}{0}, \quad \binom{1}{1}, \quad \binom{2}{0}, \quad \binom{2}{1}, \quad \binom{2}{2}, \quad \binom{3}{0}, \quad \ldots.$$

Note: For nonnegative integers n and k where $0 \le k \le n$,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

3. In the diagram, $AD : DB = 1 : 1$, $BE : EC = 1 : 2$, and $CF : FA = 1 : 3$. If the area of triangle ABC is 120, then find the area of triangle DEF.



4. Find the number of ordered triples of positive integers (a, b, c) such that $a \times b \times c = 2008^2$.

5. For a positive integer n, let $f(n)$ be the sum of the first n terms of the sequence

$$0, 1, 1, 2, 2, 3, 3, 4, 4, \ldots, r, r, r+1, r+1, \ldots.$$

For example, $f(5) = 0 + 1 + 1 + 2 + 2 = 6$.

(a) Find a formula for $f(n)$.

(b) Prove that $f(s+t) - f(s-t) = st$ for all positive integers s and t, where $s > t$.